

# A TWO-PHASE MODEL FOR MHD BLOOD FLOW IN NARROW TUBES

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Blood flow in a steady magnetic field has been of great interest over recent years. Many researchers have examined the effects of magnetic fields on velocity profiles and major studies have focused on the effects of magnetic field on the blood apparent viscosity. In this paper, the magnetohydrodynamic blood flow in a narrow tube is described using the two-layered model. The model consists of a central core region enriched with various types of blood cells and a cell-free layer surrounding the core. Exact solutions for the axial velocity, flow rate, and apparent viscosity are presented. The main result of the present study is that the effect of the magnetic field is to decrease the blood flow rate. The decrease in the flow rate is due to an increase in the apparent viscosity of the blood due to the magnetic field. The results are presented graphically and the effect of the magnetic field on the apparent viscosity is discussed and compared with reported literature values.

*Keywords:* Two-phase model; MHD blood flow; flow rate; hematocrit; apparent viscosity.

## 1. Introduction

The theoretical studies of blood flow phenomena are very useful for the diagnosis of a number of cardiovascular diseases in human physiology and for other clinical purposes. The movement of blood in an externally applied magnetic field is governed by the laws of magnetohydrodynamics (MHD). When the body is subjected to a magnetic field the charged particles of the blood flowing transversally to the field get deflected by the Lorentz force, thus including electrical currents and voltages across the vessel walls and in the surrounding tissues strong enough to be detected at the surface of the thorax in the electrocardiogram. Therefore, the interactions between these induced currents and applied magnetic field can cause a reduction of blood flow rate.

Magnetic field interactions with blood flow have been demonstrated by many authors. Korchevskii and Marochnik<sup>1</sup> first proposed a velocity profile solution for blood flow as a Newtonian fluid between two parallel plates under a constant pressure gradient with a perpendicular magnetic field. Other studies focused on flow in a rigid circular tube with non-conducting walls placed in a transverse magnetic field to offer a more realistic model for blood flow in vessels. In this case, the most complete solution of the MHD equations was proposed by Gold<sup>2</sup> under a constant pressure gradient. Vardanyan<sup>3</sup> derived an approximate steady solution where velocity profile and flow rate were calculated by neglecting the induced fields. More recent studies were based on these founding works, such as Keltner et al.<sup>4</sup> where a comparison was established between the results of Gold and Vardanyan to assess the consequences of neglecting the inductions.

In biophysics, blood flow is considered to be a two-phase flow where cells form a suspension in a liquid, the blood plasma<sup>5</sup>. In larger vessels the nature of blood is almost

homogenous and we may readily compute the MHD effects using parabolic (i.e., Poiseuille) velocity distribution<sup>6</sup>. But for flow in narrow vessels, the blood cells tend to migrate along the centre of the vessel which give rise to a core region in which the concentration of blood cells is high, and a slower moving peripheral plasma region which is almost cell free<sup>7,8</sup>. So the flow of blood in a narrow vessel may be explained by a two-layered model<sup>9-11</sup>. A good number of models have been developed regarding this phenomenon<sup>12,13</sup>.

The tube hematocrit is defined as the ratio of RBC volume in a vessel to the total volume of this vessel. The discharge hematocrit is defined as the ratio of RBC flow to the whole blood flow rate. For large vessels, RBC diameter is small compared to the tube diameter, and tube and discharge hematocrits tend to be equal. For smaller vessels, RBCs have a tendency to concentrate in the center of the vessel creating a plasma boundary near the vessel wall thus decreasing the tube hematocrit compared to the discharge hematocrit. For capillaries, when the RBC occupies almost the whole vessel section then RBC and plasma have approximately the same velocity. In this case, tube and discharge hematocrits are similar.

Paries et al. derived empirical relationships of the relative apparent viscosity and mean tube hematocrit from in vitro<sup>14,15</sup> and in vivo<sup>16</sup>. Nair et al.<sup>17</sup> used a two-phase model for the blood in modeling transport of oxygen in the arterioles. They considered a cell-rich core surrounded by a cell-free plasma layer. In the cell-rich core, the radial hematocrit distribution was expressed as a power law profile with maximum at the center of the tube. Seshadri and Jaffrin<sup>18</sup> modeled the outer layer as cell-depleted, having a lower hematocrit than in the core. The concentration of RBCs in the cell-depleted layer was assumed to be 50% of that in the core. Gupta et al.<sup>19</sup> divided the outer layer into a cell-free plasma layer and cell-depleted layer. In both these studies, the velocity profiles in the core was assumed to follow a power law.

Including some recent studies, a number of investigations have been conducted in the literature using particulate suspension theory to describe the flow of blood in small vessels. Srivastava and Srivastava<sup>20</sup> proposed a two-phase model to address pulsatile blood flow in the entrance region of an artery. Srivastava et al.<sup>21</sup> applied the theory to study the effects of an external body acceleration on blood flow through small diameter tubes which Srivastava<sup>22,23</sup> dealt with the problem of blood flow through stenotic vessels representing blood by an erythrocytes-plasma suspension.

From this survey it is clear that, this problem being more significant for applications in biotechnology. Up to date, no one has obtained the analytical form of the magnetic field effects on the blood apparent viscosity. The purpose of this paper is therefore to investigate the effect of an external magnetic field on the blood flow in a narrow vessel. Here we consider blood as a two-layered model. The model consists of a core region, enriched with various types of blood cells and a cell-free peripheral plasma layer. The velocity, flow rate, and the blood apparent viscosity are determined. The effect of magnetic field and thickness of core region on them are obtained analytically.

## 2. Governing Equations and Boundary Conditions

Consider the motion of blood as an electrically conducting fluid in a circular tube of radius  $R$  and length  $L$  in the presence of a magnetic field acting along the radius of the tube. Blood is represented by a two-fluid model consisting of a core region of radius  $r_i$  and a cell-free layer outside the core containing plasma with an effective viscosity  $\mu_o$ . The core region contains a suspension of red blood cells in plasma with an effective viscosity  $\mu_i$  ( $\mu_i > \mu_o$ ). Thus, this model consists of a cylindrical core surrounded by a less viscous annular cell-free layer, see Figure 1.

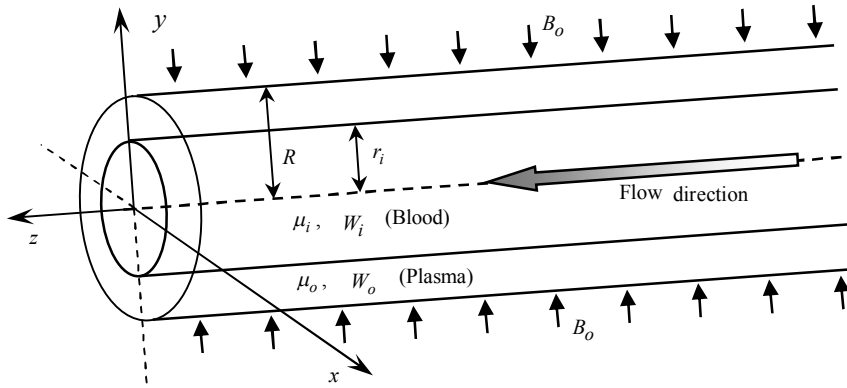


Fig. 1. Flow model geometry

The steady flow of blood as a Newtonian incompressible fluid in the presence of a magnetic field is governed by the conservation laws of mass and of momentum which are:

$$\nabla \cdot \underline{V} = 0, \quad (1)$$

$$\rho(\underline{V} \cdot \nabla \underline{V}) = -\nabla p + \mu \nabla^2 \underline{V} + \underline{f}, \quad (2)$$

where  $\underline{V}$  is the velocity vector,  $\rho$  is the density,  $\nabla p$  is the pressure gradient and  $\underline{f}$  is the body force per unit volume. In the presence of magnetic field, the magnetic body force takes the form:

$$\underline{f} = \underline{J} \times \underline{B}, \quad (3)$$

where  $\underline{J}$  is the electric current density and  $\underline{B}$  is the total magnetic field vector. The current density may be expressed by the generalized Ohm's law as:

$$\underline{J} = \sigma(\underline{E} + \underline{V} \times \underline{B}), \quad (4)$$

where the terms  $\sigma \underline{E}$  and  $\sigma(\underline{V} \times \underline{B})$ , respectively, represent the conduction and induction currents and  $\sigma$  is the electrical conductivity. In the low magnetic Reynolds number approximation<sup>24</sup>, in which the induced magnetic field can be ignored, the magnetic body

force becomes  $\sigma(\underline{V} \times \underline{B}) \times \underline{B}$  when imposed and induced electric fields are negligible and only the magnetic field  $\underline{B}$  contributes to the current  $\underline{J} = \sigma(\underline{V} \times \underline{B})$ .

Let us consider a cylindrical coordinate system  $(r, \theta, z)$  having the  $z$ -axis coincident with the vessel axis. By assuming that the flow is unidirectional and axisymmetric with no swirl in a circular rigid vessel, its velocity can be written as  $\underline{V} = [0, 0, W(r)]$  which satisfies the equation of continuity, Eq. (1), identically. In such hypothesis the convection term,  $(\underline{V} \cdot \nabla \underline{V})$ , in Eq. (2) vanishes and the pressure is thus a function of the position  $z$  only. In view of these assumptions, the flow would be governed by the equation:

$$-\nabla p + \mu \nabla^2 \underline{V} + \sigma(\underline{V} \times \underline{B}) \times \underline{B} = 0. \quad (5)$$

Let us consider  $W_i$  and  $W_o$  be the velocity of blood in the core and plasma layers respectively along the  $z$ -direction. The external constant magnetic field is applied in the direction of vessel radius such as  $\underline{B} = B_o \hat{r}$  and the vessel is considered to have non-conducting walls. Taking into account these assumptions, the velocity profile can be solely defined by the  $z$ -component of the momentum equation, Eq. (5), as:

$$-\frac{dp}{dz} + \mu_i \left( \frac{d^2 W_i}{dr^2} + \frac{1}{r} \frac{dW_i}{dr} \right) - \sigma B_o^2 W_i = 0; \quad 0 \leq r \leq r_i, \quad (6)$$

for the core layer with red blood cells and

$$-\frac{dp}{dz} + \mu_o \left( \frac{d^2 W_o}{dr^2} + \frac{1}{r} \frac{dW_o}{dr} \right) = 0; \quad r_i \leq r \leq R, \quad (7)$$

for the plasma layer.

The boundary conditions are the standard no-slip conditions of velocities and shear stresses at the vessel wall and the interface of two layers. These conditions are given as:

$$W_o = 0, \quad \text{at} \quad r = R, \quad (8)$$

$$W_o = W_i, \quad \text{and} \quad \tau_o = \tau_i, \quad \text{at} \quad r = r_i, \quad (9)$$

$$\frac{dW_i}{dr} = 0, \quad \text{at} \quad r = 0, \quad (10)$$

where  $\tau_o = \mu_o \frac{dW_o}{dr}$  and  $\tau_i = \mu_i \frac{dW_i}{dr}$  are the shear stress of the plasma and core layers, respectively. Equation (10) shows that the core velocity  $W_i$  is bounded on the  $z$ -axis and because of symmetry, the velocity gradient vanishes along the axis of the vessel.

### 3. Solution of the Problem

The solution of the problem requires the determination of the functions  $W_i$  and  $W_o$  from Eqs. (6) and (7) such that the boundary conditions, Eqs. (8) - (10), are satisfied. To do this, let us introduce the following change of variables:

$$\eta = \frac{r}{R}, \quad \alpha = \frac{r_i}{R}, \quad (11)$$

in which  $\alpha$  represents the dimensionless core radius. When  $\alpha = 0$ , the core layer is absent and the blood is a homogenous Newtonian fluid. When  $\alpha$  increases ( $\alpha \rightarrow 1$ ), the core layer become increasingly important which equivalent to blood flow in a large tube.

In terms of the new variables given in Eq. (11), the governing equations, Eqs. (6) and (7), becomes:

$$\frac{R^2}{\mu_i} \left( -\frac{dp}{dz} \right) + \frac{d^2 W_i}{d\eta^2} + \frac{1}{\eta} \frac{dW_i}{d\eta} - m W_i = 0; \quad 0 \leq \eta \leq \alpha, \quad (12)$$

for the core region and

$$\frac{R^2}{\mu_o} \left( -\frac{dp}{dz} \right) + \frac{d^2 W_o}{d\eta^2} + \frac{1}{\eta} \frac{dW_o}{d\eta} = 0; \quad \alpha \leq \eta \leq 1, \quad (13)$$

for the plasma region. In Eq. (12), the symbol  $m$  represents the magnetic parameter (Stuart number) defined by:

$$m = \frac{\sigma B_o^2 R^2}{\mu_i}, \quad (14)$$

the magnitude of  $m$  is an index to the relative importance of magnetic forces. When  $m = 0$ , magnetic forces are absent; when  $m$  increases, the magnetic forces become increasingly important.

The boundary conditions imposed on the functions  $W_i$  and  $W_o$  in terms of the new variables are:

$$W_o = 0, \quad \text{at} \quad \eta = 1, \quad (15)$$

$$W_o = W_i, \quad \text{and} \quad \mu_o \frac{dW_o}{d\eta} = \mu_i \frac{dW_i}{d\eta}, \quad \text{at} \quad \eta = \alpha, \quad (16)$$

$$\frac{dW_i}{d\eta} = 0, \quad \text{at} \quad \eta = 0. \quad (17)$$

The expressions for the velocities  $W_i$  and  $W_o$  obtained as the solutions of Eqs. (12) and (13), subject to the boundary conditions (15) – (17), are given as:

$$W_i = \frac{R^2}{\mu_o} \left( -\frac{dp}{dz} \right) \left\{ \left( \frac{1}{4} (1 - \alpha^2) - \frac{\mu_o}{m\mu_i} \right) \frac{I_o(\sqrt{m}\eta)}{I_o(\sqrt{m}\alpha)} + \frac{\mu_o}{m\mu_i} \right\}, \quad 0 \leq \eta \leq \alpha, \quad (18)$$

for the core layer

$$W_o = \frac{R^2}{4\mu_o} \left( -\frac{dp}{dz} \right) (1 - \eta^2), \quad \alpha \leq \eta \leq 1, \quad (19)$$

for the plasma layer, where  $I_o(x)$  is the modified Bessel function of zero-order.

#### 4. Flow Rate

The volumetric flow rate of blood is equal to the sum of the flow rates across the two regions<sup>10, 25</sup> (core and plasma layers). So,

$$\bar{Q} = 2\pi R^2 \int_0^\alpha W_i \eta \, d\eta + 2\pi R^2 \int_\alpha^1 W_o \eta \, d\eta, \quad (20)$$

where the first and the second terms in the last equation represent respectively the flow rates across the core and the plasma regions. By using the velocity functions from Eq. (18) and (19), the integral in Eq. (20) can be calculated. Therefore, the rate of blood flow,  $\bar{Q}$ , in the vessel is now calculated as:

$$\bar{Q} = \frac{\pi R^4}{\mu_o} \left( -\frac{dp}{dz} \right) \left\{ \frac{2\alpha I_1(\sqrt{m}\alpha)}{\sqrt{m} I_o(\sqrt{m}\alpha)} \left[ \frac{1}{4}(1-\alpha^2) - \frac{\mu_o}{m\mu_i} \right] + \frac{\alpha^2 \mu_o}{m\mu_i} + \frac{1}{8}(1-\alpha^2)^2 \right\}. \quad (21)$$

## 5. Asymptotic Solutions

In order to discuss the results of the theoretical model proposed in the study an asymptotic solutions are used to evaluate the analytical results for velocity profiles, flow rate and apparent viscosity obtained in Eqs. (18)-(20) for various values of the parameters involved.

### 5.1. Velocity function

By using the standard approximation formula of Bessel function  $I_o(x) = \sum_{n=0}^{\infty} \frac{1}{n^2!} \left(\frac{1}{2}x\right)^{2n}$ , the velocity function in the core can be expressed in the following parabolic distribution:

$$W_i = W_{\max} (1 - B\eta^2), \quad \alpha \leq \eta \leq 1, \quad (22)$$

where

$$W_{\max} = \frac{R^2 (-dp/dz)}{\mu_o (4 + m\alpha^2)} \left[ 1 - \alpha^2 \left( 1 - \frac{\mu_o}{\mu_i} \right) \right], \quad B = \frac{4 \frac{\mu_o}{\mu_i} - m(1-\alpha^2)}{4 \left[ 1 - \alpha^2 \left( 1 - \frac{\mu_o}{\mu_i} \right) \right]}. \quad (23)$$

The parameter  $B$  represents the deviation from parabolic flow. When  $m=0$  and  $\mu_i \rightarrow \mu_o$ ,  $B \rightarrow 1$  and the velocity profile becomes parabolic throughout the entire cross section of the tube. Notice that at  $m=0$  (where there is no magnetic field), Eq. (22) gives the same form of Sharan et al.<sup>11</sup>.

### 5.2. Flow rate

Since  $I_1(x) = \sum_{n=0}^{\infty} \frac{1}{n!(n+1)!} \left(\frac{1}{2}x\right)^{2n+1}$ , it follows that the rate of flow, Eq. (21), approaches the following form:

$$\bar{Q} = \frac{\pi R^4}{8\mu_o (4 + m\alpha^2)} \left( -\frac{dp}{dz} \right) \left[ 4 \left( 1 - \alpha^4 + \alpha^4 \frac{\mu_o}{\mu_i} \right) + m\alpha^2 (1 - \alpha^2) \right]. \quad (24)$$

### 5.3. Apparent magnetic viscosity of blood

Equation (24) can be written as:

$$\bar{Q} = \frac{\pi R^4}{8\bar{\mu}_{app}} \left( -\frac{dp}{dz} \right), \quad (25)$$

where  $\bar{\mu}_{app}$  is the magnetic apparent (effective) blood viscosity which is a measure of viscosity when the magnetic field is applied. The comparison between Eqs. (24) and (25) gives:

$$\bar{\mu}_{app} = \frac{\mu_o(4+m\alpha^2)}{4\left(1-\alpha^4 + \alpha^4 \frac{\mu_o}{\mu_i}\right) + m\alpha^2(1-\alpha^2)}, \quad (26)$$

when  $m=0$  and  $\alpha \rightarrow 1$ ,  $\bar{\mu}_{app} \rightarrow \mu_i$ , which corresponds to the viscosity of blood as a single layered model. In addition, when  $m=0$ , one obtains the same expression for the apparent viscosity derived from steady Newtonian fluid model of Bugliarello and Sevilla (1970)<sup>25</sup> as:

$$\mu_{app} = \frac{\mu_o}{1-\alpha^4 + \alpha^4 \frac{\mu_o}{\mu_i}}, \quad (27)$$

comparing the magnetic apparent viscosity  $\bar{\mu}_{app}$  to the apparent viscosity  $\mu_{app}$  in the absence of a magnetic field, we obtain the ratio

$$\tilde{\mu} = \frac{\bar{\mu}_{app}}{\mu_{app}} = \frac{(4+m\alpha^4)(1-\alpha^2 + \alpha^2 \frac{\mu_o}{\mu_i})}{m\alpha^2(1-\alpha^2) + 4(1-\alpha^4 + \alpha^4 \frac{\mu_o}{\mu_i})}. \quad (28)$$

Clearly, the presence of a magnetic field increases the blood viscosity and hence diminishes the flow rate. Equation (27) represents the same expression derived by Sharan et al.<sup>11</sup> and recovers the result obtained in Haynes (1960)<sup>10</sup> when  $\mu_o = 1 \text{ cp}$ .

## 6. Results and Discussion

The MHD flow mechanics of blood in narrow tubes is characterized in this model by several parameters: the dimensionless core radius  $\alpha$ , the dimensionless cell-free layer thickness  $(1-\alpha)$ , the magnetic parameter  $m$  and the rheological parameters of the fluids, i.e., the viscosities of the core  $\mu_i$  and outer layer  $\mu_o$ . The thickness of the cell-free layer (plasma) has been measured in vitro and in vivo. Yamaguchi and Yamakura<sup>26</sup> measured the cell-free layer thickness in cerebral microvessels and found that the ratio of the cell-free layer thickness to the vessel radius in venues varies between 0.14 to 0.36. Tateishi et al.<sup>27</sup> measured the plasma layer thickness for different values of the systemic hematocrit and the vessel size in microvessels of isolated rabbit mesentery. They found substantial reduction in the values of the cell-free layer thickness for increasing values of

hematocrit. For microvessels of inner diameter 30 to 40  $\mu m$ , the cell-free layer thickness varied between 6  $\mu m$  and 1.5  $\mu m$  as the hematocrit was increased from 8 to 50%.

Because the goal of this study is to investigate the effect of an external magnetic field on the normal blood flow in a narrow vessel, we have chosen parameters of the flow from the study of Sharan et al.<sup>11</sup> and Srivastava<sup>28</sup>. The radius  $R$  of the vessel is chosen as  $R = 70 \mu m$ . The values of the magnetic parameter  $m$  are chosen between 0 and 4. The dimensionless core layer thickness  $\alpha$  is chosen as  $\alpha = 0.92$ ; in some calculations,  $\alpha$  is varied between 0 and 1. The viscosities of the core  $\mu_i = 4.03 cp$  and outer layer  $\mu_o = 1.24 cp$ .

The axial velocity profiles ( $W_i$  and  $W_o$ ) computed from the present study, Eqs. (18) and (19) respectively, are displayed graphically in Figures 2 and 3 for two values of the pressure gradients,  $-dp/dz = 76 \text{ dyne/mm}^3$  and  $-dp/dz = 67.5 \text{ dyne/mm}^3$ . The effect of the magnetic field parameter  $m$  is also shown in that figures. In those figures we observe that the inner core velocity decreases with the increase in the value of  $m$ . In Figure 4, the nature of velocity profile is shown for  $\alpha = 0.85$ . The comparison between Figures 3 and 4 shows that the velocity profile decreases with increase in  $\alpha$ . From Figures 2 to 4, one observes that core velocity at the tube axis assumes higher magnitude than the plasma velocity but the difference in their magnitudes decreases with increasing the dimensionless radial coordinate  $\eta$  towards the interface and at the interface the plasma velocity coincidences with the blood velocity as indicated in the boundary conditions, Eq. (16).

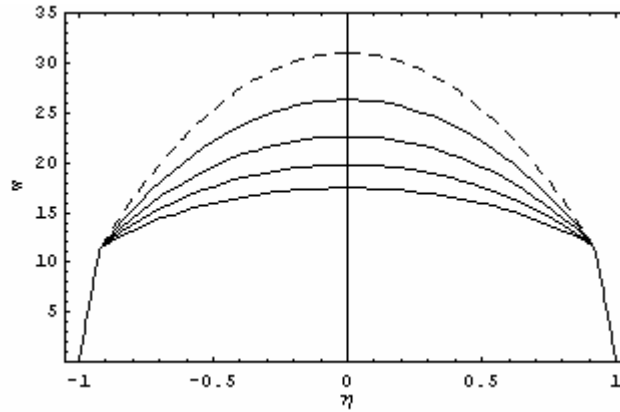


Fig. 2. Velocity profiles at  $\alpha = 0.92$ ,  $-dp/dz = 76 \text{ dyne/mm}^3$  and magnetic parameter  $m = 0, 1, 2, 3, 4$  (top to bottom)

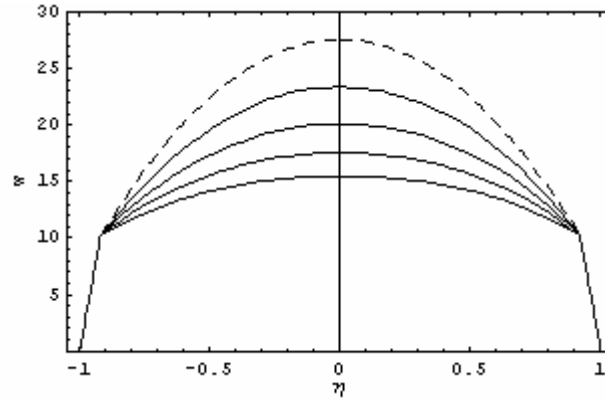


Fig. 3. Velocity profiles at  $\alpha = 0.92$ ,  $-dp/dz = 67.5 \text{ dyne/mm}^3$  and magnetic parameter  $m = 0, 1, 2, 3, 4$  (top to bottom)

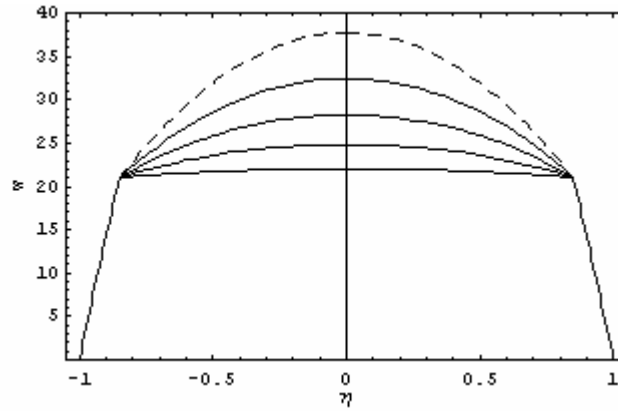


Fig. 4. Velocity profiles at  $\alpha = 0.85$ ,  $-dp/dz = 67.5 \text{ dyne/mm}^3$  and magnetic parameter  $m = 0, 1, 2, 3, 4$  (top to bottom)

The parameter  $B$  in Eq. (22) is the bluntness velocity profile in the core region. For a given fluid, this parameter depends on the dimensionless core radius  $\alpha$ , the magnetic parameter  $m$  and viscosities of the core  $\mu_i$  and outer layer  $\mu_o$ . The velocity profile becomes parabolic when  $B \rightarrow 1$ . Figure 5 shows that the bluntness parameter  $B$  increases as  $\alpha$  increases. In other words, Figure 6 shows that the  $B$  decreases as  $m$  increases. Thus, the velocity profile becomes more parabolic when  $\alpha$  is increased or  $m$  is decreased. A number of experimental studies *in vitro*<sup>25</sup> and *in vivo*<sup>26</sup> have reported that the velocity profiles were blunted and become more parabolic with increased tube diameter.

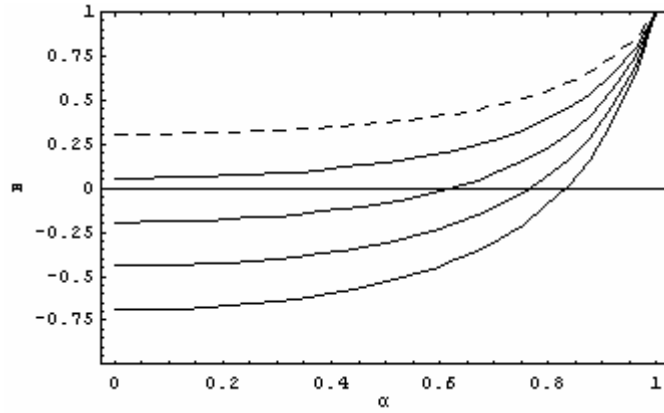


Fig. 5. Variation of bluntness parameter ( $B$ ) with  $\alpha$  at  $m = 0, 1, 2, 3, 4$  (top to bottom)

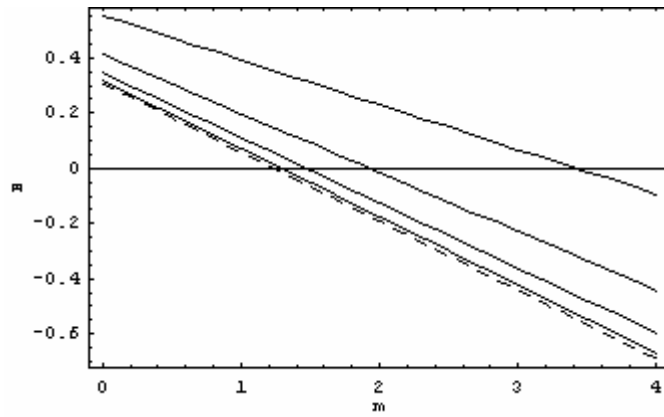


Fig. 6. Variation of bluntness parameter  $B$  with  $m$  at  $\alpha = 0, 0.2, 0.4, 0.6, 0.8$  (bottom to top)

The values of the volumetric flow rate  $\bar{Q}$  are calculated using Eq. (24). The variation of  $\bar{Q}$  against  $-dp/dz$  is shown on Figure 7 for different values of  $m$  taking  $\alpha = 0.92$ . Here we observe that the flow rate  $\bar{Q}$  is affected by the magnetic field parameter  $m$ . Figure 8 shows that the presence of a magnetic field diminishes the flow rate.

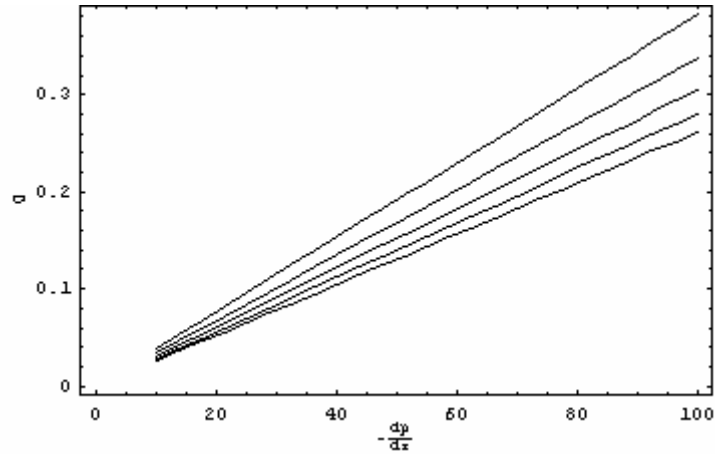


Fig.7. Pressure – flow rate relationship at  $\alpha = 0.92$  and  $m = 0, 1, 2, 3, 4$  (top to bottom)

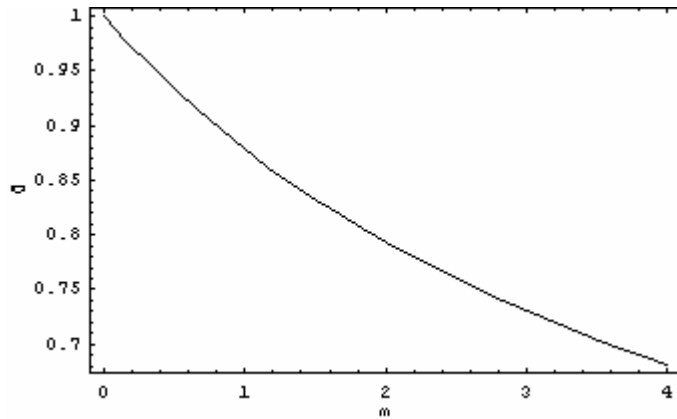


Fig.8. Decrease of the flow rate in the presence of magnetic field at  $\alpha = 0.92$ .

The ratio of the magnetic apparent viscosity under the influence of magnetic field,  $\bar{\mu}_{app}$ , to the apparent viscosity in the absence of magnetic field,  $\mu_{app}$ , is plotted versus the magnetic parameter  $m$  in Figure 9. It is clear that there is an increase in the magnetic apparent viscosity for the blood under the influence of the magnetic field. This behavior be related to the existence of the magnetic torque, which will cause the red blood cells to orient. Notice that, when the applied magnetic field reaches zero ( $m = 0$ ) the ratio  $\frac{\bar{\mu}_{app}}{\mu_{app}} = 1$ .

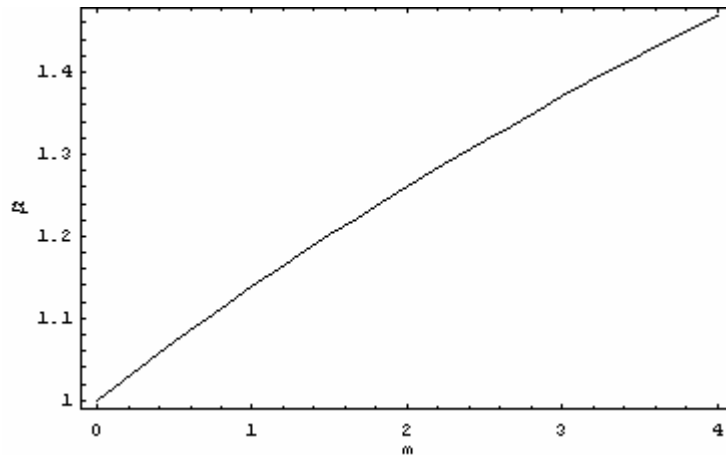


Fig.9. Increase of the apparent blood viscosity in the presence of magnetic field at  $\alpha = 0.92$ .

## 7. Conclusions

In biological systems, the MHD blood flow is of great importance. In this paper, a two-phase model for the MHD blood flow in the narrow vessels is described. The governing differential equations are solved analytically. The effect of the magnetic field parameter on the velocity field, flow rate and apparent viscosity of blood is studied. It is found that increasing the MHD – parameter decreases the blood velocity and flow rate. Therefore, a remarkable phenomenon is that by using an external magnetic field we can regulate the blood flow.

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